Model Tracking Dual Stochastic Controller Design Under Irregular Internal Noises

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Although many methods about the control of irregular external noise have been introduced and implemented, it is still necessary to design a controller that will be more effective and efficient methods to exclude for various noises. Accumulation of errors due to model tracking, internal noises (thermal noise, shot noise and $1/f$ noise) that come from elements such as resistor, diode and transistor etc. in the circuit system and numerical errors due to digital process often destabilize the system and reduce the system performance. New stochastic controller is adopted to remove those noises using conventional controller simultaneously. Design method of a model tracking dual controller is proposed to improve the stability of system while removing external and internal noises. In the study, design process of the model tracking dual stochastic controller is introduced that improves system performance and guarantees robustness under irregular internal noises which can be created internally. The model tracking dual stochastic controller utilizing F-P-K stochastic control technique developed earlier is implemented to reveal its performance via simulation.

Key Words : Internal Noise, Model Tracking Dual Stochastic Controller, Power Spectral Density, F-P-K Method, Error Suppression

1. Introduction

There are various ways for improving performance of system that design control system and correct stability. Much research has been done about dual control techniques to achieve these purposes simultaneously. Normally dual control technique is used to improve control performance and stability of system. (Knohl et al., 2003) Lot of study have been carried out to design dual controller with a neural network and adaptive method. (Allison et al., 1995 ; Fabri, and Kandirkamanathan, 1998 ; Filatov and Unbehauen, 2000 ; EDISON TSE and YAAKOV BAR-SHALOM, 1973) However, numerical errors are generated while implementing the control technique in digital manner have

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never been dealt with. Also there are internal noises that could arise in the system inside, thermal noises from resistance of inside circuit, shot noises from the diode and the transistor, and the noises of by a frequency ingredient.

Basic characteristics of those noises created from numerical process and system interior are random in nature. As well as irregular noise increases fatigue level in control system, it can become input of impulse form in amplification circuit that does to amplify the value of control gains by reiteration of irregular signal. In this paper, it is proposed to design a controller that suppress those irregular internal noises and improve the efficiency and stability of the system simultaneously assuming that those errors and noises are random. The dual control method embodies the controller, which follows input model, suppresses the error that could be created while following model and suppresses the noise from the system inside which is irregular. To solve this problem effectively, stochastic controller is used to suppress internal noises that are basically random. The stochastic controller, which is different from existing conventional stochastic controller, analyzes the system information of random noise in stochastic domain. The proposed control technique designed in stochastic domain is a method to suppress random noise. In order to design stochastic controller, system dynamic model in time domain is transformed to dynamic moment equation in stochastic domain by F-P-K (Fokker-Planck-Kolmogorov) approach. Through a F-P-K method, the information of random noise in time domain are transformed to a constant PSD (Power Spectral Density) and a controller is designed in stochastic domain. Constant control gain obtained in stochastic domain is utilized to generate a physical control signal in time domain. Random signal generating algorithm such as a Monte-Carlo method is adopted for the purpose. (Heo et al., 2002 ; 2003 ; Kim et al., 1999 ; Lee et al., 2005) The study reveals remarkable performance of proposed dual stochastic controller operating under various random noises through simulation.

2. Dual Stochastic Controller Design

2.1 Information of internal noise

Irregular noises are generated from several sources. Firstly, it is considered that numerical error is generated in using of digital control technique. At the best of authors' knowledge, no research for the compensation of numerical errors accumulated during digital operation in conventional controller has been reported. Various noises could be generated from system inside, that consist of thermal noise, shot noise and $1/f$ noise. Those internal noises from system circuit are shown in next three sources. (Howard, 2002)

Thermal noise has a zero mean and statistical arguments can be used to show that the power spectral density of the processes is

$$G_R(f)=\frac{2h|f|R}{e^{h|f|/kT}-1} \tag{1}$$

where T is the absolute temperature, k is Boltzmann's constant (1.38×10^{-23} J/K), h is Planck's constant (6.62×10^{-34} J·sec) and R is the resistance of the material.

For frequencies, such that $|f|<0.1kT/h\approx10^{12}$ Hz (assuming $T=300$K) a Taylor series expansion for the exponential term in these equations,

$$e^{h|f|/kT}\approx1+h|f|/kT \tag{2}$$

is valid, and the following approximations hold

$$G_R\approx2kTR \tag{3}$$

Shot noise is associated with charge carriers crossing a barrier, such as that inherent in a PN junction, at random times, but with a constant average rate. The power spectral density, for all but high frequencies, is given by

$$G_S(f)\approx q\bar{I}+\bar{I}^2\delta(f) \tag{4}$$

where q is the electronic charge ($1.6\times10^{-19}C$), and $\bar{I}$ is the mean current.

Ignoring the impulse at zero frequency, the power spectral density of shot noises is given by

$$G_S(f)\approx q\bar{I} \tag{5}$$

The $1/f$ noise is defined as following. The power spectral density of a $1/f$ random process has a power spectral density given by

$$G_f(f) = \frac{k}{f^{\alpha}} \tag{6}$$

where k is a constant, and α is close to unity.

The power spectral density of $1/f$ noises has a $1/f$ form at low frequencies, and at higher frequencies is constant.

However, as a first attempt in the series of research all those noises are assumed to be white noise in the study and controller that could suppress the white noise is designed in stochastic domain. In addition to conventional model tracking controller, dual stochastic controller is designed to follow the control input and suppress the internal noises.

2.2 Model tracking controller design

Model tracking controller is designed as follows. Consider a general second order system, which is shown in Fig. 1. As a practical example, general model tracking controller using conventional PI controller is designed.

Where, $r(s)$ is a input reference, $d(s)$ is a external disturbance and $g(s)$ is a transfer function of system.

Control gain for PI (Proportion-Integration) compensator is given by Eq. (7)

$$K(s) = K_p + K_i \frac{1}{s} \tag{7}$$

where K_p is proportional gain, K_i is integration gain.

Transfer function of system is given by Eq. (8)

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{8}$$

Then the error of system applied PI controller is given by Eq. (9)

$$e(s) = \frac{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}{s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + K_p s + K_i} \tag{9}$$

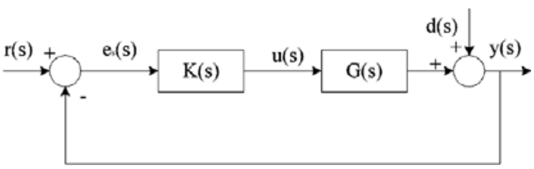


Fig. 1 Scheme of model tracking controller

No external disturbance is assumed in model tracking controller design. In section 2.2, the model-tracking error, internal noises in system inside are discussed and also stochastic controller for suppressing those noises is explained in following sections.

2.3 Stochastic controller design

In previous section, model-tracking controller for general second order system is discussed. We continue that discussion in this section by showing how stochastic controller is designed to suppress the random noise. So simple example for stochastic controller is introduced here. (Kim et al., 2005 ; Lee, 2005)

State equation of system is transformed to moment equation in stochastic domain via F-P-K method. And in stochastic domain the controller for suppressing the random noises is designed using PI control method. Then the control gain of stochastic controller in stochastic domain is transformed to random control input in time domain via Monte-Carlo method.

F-P-K method is one way of studying the behavior of a system probability density function. There are two basic assumptions for the derivation of F-P-K equation. First, random input is always sufficiently small, so that the perturbed motion can be determined by superimposing random fluctuations of first order smallness to a continuous mean trajectory. Second, the random process under consideration is a Markov process, and does not depend on its past history. The general form of the F-P-K equation is given by

$$\begin{aligned} \frac{\partial}{\partial t} p(\underline{X}, t) = & -\sum_{i=1}^{n} \frac{\partial}{\partial t} \{ a_i(\underline{X}, t) p(\underline{X}, t) \} \\ & + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2}{\partial X_i \partial X_j} \{ b_{ij}(\underline{X}, t) p(\underline{X}, t) \} \end{aligned} \tag{10}$$

where drift coefficient, $a_i(\underline{X}, t)$ and diffusion coefficient, $b_{ij}(\underline{X}, t)$ are defined respectively as Eq. (11) and Eq. (12)

$$a_i(\underline{X}, t) = \lim_{\Delta n \to 0} \frac{1}{\Delta t} E[\{ x_i(t+\Delta t) - x_i(t) \}] \tag{11}$$

$$\begin{aligned} b_{ij}(\underline{X}, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[& \{ x_i(t+\Delta t) - x_i(t) \} \\ & \{ x_j(t+\Delta t) - x_j(t) \}] \end{aligned} \tag{12}$$

The solution of these equations gives the probabilistic behavior of the system response. In many case, however, it is not possible to obtain a closed form analytical solution of the F-P-K equation, one can generate a set of differential equations for the response moments.

Let $\Phi(\underline{X})$ be a general function of the response co-ordinate vector $\underline{X}$

$$\Phi(\underline{X})=X_1^{k_1}X_2^{k_2}\cdots X_n^{k_n}=\prod_{i=1}^{n}X_i^{k_i} \tag{13}$$

Such that the following notation expresses the moments of order k_i

$$\begin{aligned} m_{k_i,k_2,\cdots,k_n} &= E[\Phi(\underline{X})] \\ &= \int\cdots\int_{-\infty}^{\infty}\Phi(\underline{X})\,p(X,t)\,dX_1dX_2\cdots dX_n \end{aligned} \tag{14}$$

As an example, a second order system is considered.

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2x=f(t) \tag{15}$$

where $f(t)$ is white noise with zero mean, and Gaussian distribution.

Eq. (15) can be written in the Ito's stochastic differential Eq. (17)

$$x=X_1,\ \dot{x}=X_2 \tag{16}$$

$$\begin{aligned} dX_1 &= X_2dt \\ dX_2 &= \{-\omega_n^2X_1-2\zeta\omega_nX_2\}\,dt \end{aligned} \tag{17}$$

The evolution of transitional joint probability density function of the response co-ordinates $p(\underline{X}, t)$ can be described by the F-P-K Eq. (10).

The general differential equation for moment is

$$\frac{\partial}{\partial t}E[X_1^iX_2^j]=\iint_{-\infty}^{\infty}X_1^i\frac{\partial}{\partial t}p(\underline{X},t)\,dX_1dX_2 \tag{18}$$

Thus the first order moment equations and the second order moment equations are obtained as Eq. (19).

$$\begin{aligned} \dot{m}_{10} &= m_{01} \\ \dot{m}_{01} &= -2\zeta\omega_nm_{01}-\omega_n^2m_{10} \\ \dot{m}_{11} &= -2\zeta\omega_nm_{11}-\omega_n^2m_{20} \\ \dot{m}_{20} &= 2m_{11} \\ \dot{m}_{02} &= -2\omega_n^2m_{11}-4\zeta\omega_nm_{02}+D \end{aligned} \tag{19}$$

The general form of moment equation is given by

$$\underline{\dot{m}}=A_m\underline{m}+P_mD_z+B_mD_v \tag{20}$$

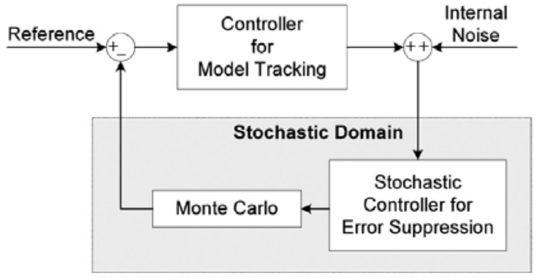


Fig. 2 Scheme of model tracking dual stochastic controller

where, D_z is PSD of noise, D_v is PSD of control input.

2.4 Dual stochastic controller design

We have discussed the design of model tracking controller to follow the control law and stochastic controller to suppress the random noises. Model tracking dual stochastic controller is designed and its basic scheme is shown in Fig. 2. The main algorithm of each controller is designed using same method described in previous section. In this section, only internal noises and numerical error are considered. In chapter 3, system with model tracking dual stochastic controller will be simulated, also the comparison will be made between the performances of the system with conventional dual controller and model tracking dual stochastic controller for the same system.

In designing a controller in stochastic domain, most of the controller design techniques used in time domain can be applied. In the study, PI controller is designed in stochastic domain and applied. Finally, the obtained control signal (in PSD value) in stochastic domain is transformed to the control input in physical time domain. The random signal algorithm such as Monte-Carlo method is used to generate the control input signal.

3. Simulation

In the preceding chapter, a model tracking dual stochastic controller is designed and numerical simulation is conducted to verify the proposed concept and the performance of the proposed model tracking dual stochastic controller. Arbitrary types of pulse signals are used as an input reference to follow as is in Fig. 3. Also basic characteristics of internal noises in use are shown in Fig. 4.

The performance of proposed controller is compared with conventional model tracking controller such as dual PI (Proportional-Integrate) controller. The proposed controller consists of stochastic controller for suppressing the noise and PI (Proportional Integrate) controller for tracking the control law. In order to compare the performance of proposed dual stochastic controller, as a reference, two conventional PI controllers are used in parallel manner to suppress the noise and to track the model. As a results time response, error response and mean square response of the system with proposed dual stochastic controller and conventional controller are shown in Figs. 5～7. As is shown in Fig. 5, as far as time response is concerned proposed model tracking dual stochastic controller exhibits much better performance than conventional model tracking dual controller. Also regarding the model tracking error, new model tracking dual stochastic controller shows excellent performance as is in Fig. 6. Comparison of system mean square responses due to those controllers is made to reveal its outstanding performance in Fig. 7.

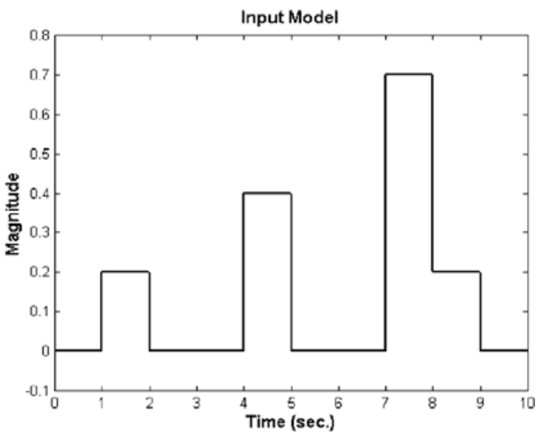
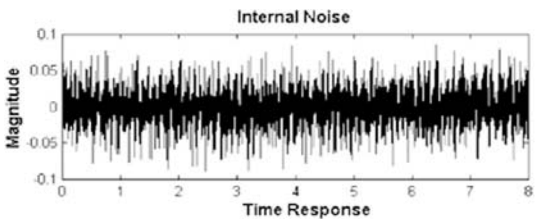
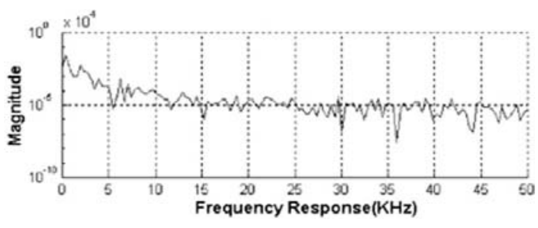


Fig. 3 Input reference

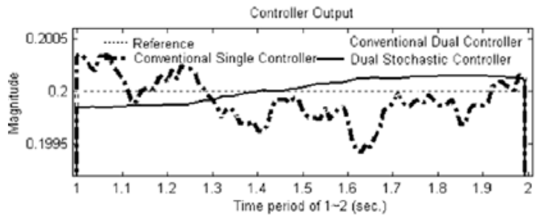


(a) Internal noise in time domain

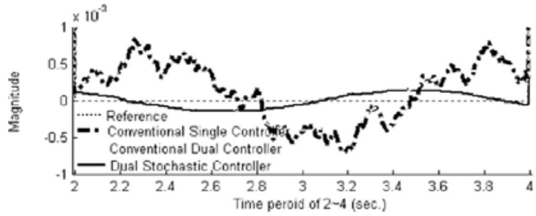


(b) Internal noise in frequency domain

Fig. 4 Information of internal noise

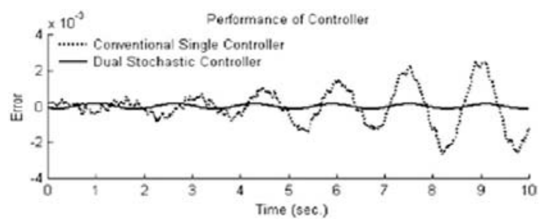


(a) Time period of 1～2 sec

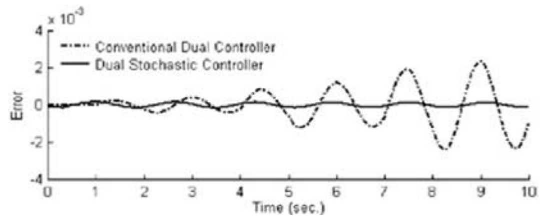


(b) Time period of 2～4 sec

Fig. 5 Time response of system



(a) Dual stochastic controller vs. Single PI Controller



(b) Dual stochastic controller vs. Dual PI Controller

Fig. 6 Comparison of errors

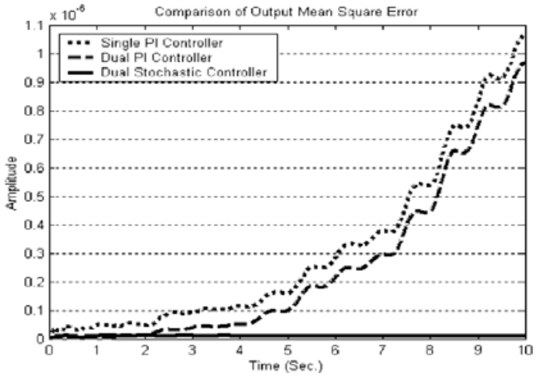


Fig. 7 Comparison of output mean square error

4. Conclusions

In this paper, "Model tracking dual stochastic controller" is proposed to suppress random internal noise and to follow the control law simultaneously. Conducted simulation reveals remarkable performance of the proposed "Model Tracking Dual Stochastic Controller" in terms of system response and is compared with conventional controller for various noises that can be created in the system. The response error of stochastic system using conventional dual controller is increased along with time and the error is accumulated, which destabilize the system. The response error of the system under internal noise when using "Model Tracking Dual Stochastic Controller" is decreased drastically, accordingly the system becomes more stable. The new concept of "Model Tracking Dual Stochastic Controller" is successfully implemented and proved to be very effective one. Also the new controller will be designed to suppress internal noises as well as random external noises and to follow the control law using dual stochastic controller technique. The newly proposed controller reveals its reliable and efficient performance via simulation and experiment is to be conducted.

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